



AD-A210 774

RESEARCH AND DEVELOPMENT TECHNICAL REPORT  
SLCET-TR-89-3

POLARIZATION MATRICES OF LITHIUM TETRABORATE

ARTHUR BALLATO

ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY

JUNE 1989

DISTRIBUTION STATEMENT

Approved for public release;  
distribution is unlimited.

**DTIC**  
**ELECTE**  
**AUG 01 1989**  
**S D**

US ARMY  
LABORATORY COMMAND  
FORT MONMOUTH, NEW JERSEY 07703-5000

89

8-01

030

## NOTICES

### Disclaimers

The citation of trade names and names of manufacturers in this report is not to be construed as official Government indorsement or approval of commercial products or services referenced herein.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE				Form Approved OMB No 0704-0188	
1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.		
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S) SLCET-TR-89-3			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
6a. NAME OF PERFORMING ORGANIZATION US Army Laboratory Command Electronics Tech & Devices Lab		6b. OFFICE SYMBOL (If applicable) SLCET-MA-A		7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State, and ZIP Code) Electronics Technology and Devices Laboratory ATTN: SLCET-MA-A Fort Monmouth, NJ 07703-5000				7b. ADDRESS (City, State, and ZIP Code)	
8a. NAME OF FUNDING / SPONSORING ORGANIZATION		8b. OFFICE SYMBOL (If applicable)		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code)				10. SOURCE OF FUNDING NUMBERS	
				PROGRAM ELEMENT NO. 1L162705	PROJECT NO. H94
				TASK NO. K9	WORK UNIT ACCESSION NO. DA303394
11. TITLE (Include Security Classification) POLARIZATION MATRICES OF LITHIUM TETRABORATE (U)					
12. PERSONAL AUTHOR(S) Arthur Ballato					
13a. TYPE OF REPORT Technical Report		13b. TIME COVERED FROM Jan 88 TO Jan 89		14. DATE OF REPORT (Year, Month, Day) 1989 June	
15. PAGE COUNT 29					
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Piezoelectric resonators; piezoelectric transducers; lithium tetraborate; acousto-optics		
09	01				
17	02				
19. ABSTRACT (Continue on reverse if necessary and identify by block number)					
<p>In analytical treatments of piezoelectric-acoustic transducers, signal processors, and resonators, the electromechanical transduction mechanism is most often expressed in terms of the elements of the piezoelectric [e] or [d] matrices. Molecular interpretations of piezoelectricity, and especially electrooptical applications, usually involve polarization as the preferred variable, and consequently the alternative [a] and [b] matrices are of interest. The elements of these latter sets are calculated for lithium tetraborate from measured elastopiezodielectric constants taken from the literature.</p>					
20. DISTRIBUTION / AVAILABILITY OF ABSTRACT <input type="checkbox"/> UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS				21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Arthur Ballato				22b. TELEPHONE (Include Area Code) (201) 544-2773	
				22c. OFFICE SYMBOL SLCET-MA-A	

# CONTENTS

	Page
INTRODUCTION . . . . .	1
CONSTITUTIVE EQUATION SETS . . . . .	1
RELATIONS AMONG MATERIAL CONSTANTS . . . . .	5
CALCULATION SEQUENCE . . . . .	7
EXPLICIT FORMULAS FOR POINT GROUP 4mm . . . . .	10
INPUT VALUES FOR LI2 B4 07 . . . . .	16
OUTPUT VALUES FOR LI2 B4 07 . . . . .	16
CONCLUSIONS . . . . .	19
REFERENCES . . . . .	20

# TABLES

Table	Page
1. Symbols, Units, and Definitions. . . . .	2
2. Relations among Material Constants . . . . .	8
3. Further Relations among Material Constants . . . . .	9
4. Elastopiezodielectric Matrices for Point Group 4mm . . . . .	11
5. Isagric Elastic Compliances . . . . .	16
6. Piezoelectric Strain Coefficients . . . . .	16
7. Dielectric Permittivities at Constant Stress . . . . .	16
8. Elastic Stiffnesses . . . . .	17
9. Elastic Compliances . . . . .	17
10. Piezoelectric [e], [h], and [a] Values . . . . .	17
11. Piezoelectric [d], [g], and [b] Values . . . . .	18

For	
RA&I	<input checked="" type="checkbox"/>
AB	<input type="checkbox"/>
ced	<input type="checkbox"/>

on/

Availability Codes	
Dist	Avail and/or Special
A-1	



Table		Page
12.	Dielectric ( $\epsilon$ s) Values . . . . .	18
13.	Dielectric ( $\chi$ i) Values . . . . .	18
14.	Dielectric ( $\beta$ et) Values . . . . .	18
15.	Dielectric ( $\zeta$ et) Values . . . . .	19

## INTRODUCTION

Electromechanical transduction taking place via the piezoelectric effect is characterized phenomenologically by constitutive equations that relate the elastic and electric variables. These equations take a variety of forms, depending upon the choice of independent and dependent variables; the choice is normally dictated by the application. For example, piezoelectric resonators in the form of thickness mode plates are most easily treated using the isagric elastic stiffnesses  $[cE]$ , the piezoelectric stress constants  $[e]$ , and the dielectric permittivities at constant strain  $[(\epsilon_s)S]$ .

Various measurement techniques yield values for the elements of a particular coefficient set more directly than those of another. The coefficients appearing in the different equation sets are, however, interrelated, so that once any one complete set is available, all the other sets of elements may be found. The most accurate and precise experimental results to date have been from plate resonator (resonance) and pulse-echo (transit-time) measurements. From the  $[cE]$ ,  $[e]$ , and  $[(\epsilon_s)S]$  matrices determined therefrom, those matrices representing material properties expressed in the other alternative forms may be calculated.

Electrooptical applications are becoming increasingly important. So also are treatments of piezoelectric and ferroelectric phenomena from the standpoint of molecular interactions. In both of these cases the constitutive equations using polarization as the independent electrical variable, rather than either electric intensity or displacement, assume greater importance than the sets traditionally used for transducer, signal processing, and resonator applications.

In this report we give the complete sets of linear constitutive equations relating elastic and electric fields. For each equation set the numerical values are computed for lithium tetraborate from the measured  $[sE]$ ,  $[d]$ , and  $[(\epsilon_s)T]$  values of Shiosaki, et al. (Ref.1). Coupling to the thermal field is neglected. Rationalized mks units are used throughout.

## CONSTITUTIVE EQUATION SETS

Symbols and units for the quantities employed are given in Table 1. In terms of these, six constitutive equation sets are used. Of these, electric intensity, dielectric displacement, and polarization each appear in two sets as an independent variable. The sets are, in compressed matrix notation, as follows. A prime denotes transpose;  $[I]$  is the unit matrix.

### I. The Piezoelectric Stress Constant Set

$$\begin{aligned} [T] &= [cE] [S] - [e]' [E] & (1) \\ [D] &= [e] [S] + [(\epsilon_s)S] [E] & (2) \end{aligned}$$

TABLE 1. SYMBOLS, UNITS, AND DEFINITIONS.

QUANTITY	UNIT	SYMBOL/DEFINITION
Elastic stress	N/m <sup>2</sup>	[T]
Elastic strain	-----	[S]
Electric intensity	V/m	[E]
Dielectric displacement	C/m <sup>2</sup>	[D]
Dielectric polarization	C/m <sup>2</sup>	[P]
Elastic compliance at constant [E], [D], [P]	m <sup>2</sup> /N	[cE], [cD], [cP]
Elastic stiffness at constant [E], [D], [P]	N/m <sup>2</sup>	[sE], [sD], [sP]
Dielectric permittivity at constant [T], [S]	F/m	[( $\epsilon$ )T], [( $\epsilon$ )S]
Dielectric constant, relative, at constant [T], [S]	-----	[( $\epsilon_r$ )T], [( $\epsilon_r$ )S] =[( $\epsilon$ )T]/( $\epsilon$ ) <sub>o</sub> , [( $\epsilon$ )S]/( $\epsilon$ ) <sub>o</sub>
Dielectric impermeability at constant [T], [S]	m/F	[( $\beta$ )T], [( $\beta$ )S] =[( $\epsilon$ )T] <sup>(-1)</sup> , [( $\epsilon$ )S] <sup>(-1)</sup>
Dielectric impermeability, relative, at constant [T], [S]	-----	[( $\beta_r$ )T], [( $\beta_r$ )S] =[( $\beta$ )T]*( $\epsilon$ ) <sub>o</sub> , [( $\beta$ )S]*( $\epsilon$ ) <sub>o</sub> =[( $\epsilon_r$ )T] <sup>(-1)</sup> , [( $\epsilon_r$ )S] <sup>(-1)</sup>
Dielectric susceptibility at constant [T], [S]	F/m	[( $\chi$ )T], [( $\chi$ )S] =[( $\epsilon_r$ )T-1]*( $\epsilon$ ) <sub>o</sub> , [( $\epsilon_r$ )S-1]*( $\epsilon$ ) <sub>o</sub>
Dielectric susceptibility, relative, at constant [T], [S]	-----	[( $\chi_r$ )T], [( $\chi_r$ )S] =[( $\chi$ )T]/( $\epsilon$ ) <sub>o</sub> , [( $\chi$ )S]/( $\epsilon$ ) <sub>o</sub>
Reciprocal dielectric susceptibility at constant [T], [S]	m/F	[( $\zeta$ )T], [( $\zeta$ )S] =[( $\chi$ )T] <sup>(-1)</sup> , [( $\chi$ )S] <sup>(-1)</sup>
Reciprocal dielectric susceptibility, relative, at constant [T], [S]	-----	[( $\zeta_r$ )T], [( $\zeta_r$ )S] =[( $\zeta$ )T]*( $\epsilon$ ) <sub>o</sub> , [( $\zeta$ )S]*( $\epsilon$ ) <sub>o</sub>
Piezoelectric stress constant	C/m <sup>2</sup>	[e]

TABLE 1. SYMBOLS, UNITS, AND DEFINITIONS. (continued)

QUANTITY	UNIT	SYMBOL/DEFINITION
Piezoelectric strain coefficient	$m/V = C/N$	[d]
Piezoelectric stress modulus	$N/C = V/m$	[h]
Piezoelectric strain constant	$m^2/C$	[g]
Piezoelectric polarization modulus	$V/m = N/C$	[a]
Piezoelectric polarization constant	$m^2/C$	[b]

Note: Square brackets, sic: [ ], denote matrices.

## II. The Piezoelectric Strain Coefficient Set

$$[S] = [sE] [T] + [d]' [E] \quad (3)$$

$$[D] = [d] [T] + [(\epsilon)s]T [E] \quad (4)$$

## III. The Piezoelectric Stress Modulus Set

$$[T] = [cD] [S] - [h]' [D] \quad (5)$$

$$[E] = -[h] [S] + [(\beta)S] [D] \quad (6)$$

## IV. The Piezoelectric Strain Constant Set

$$[S] = [sD] [T] + [g]' [D] \quad (7)$$

$$[E] = -[g] [T] + [(\beta)T] [D] \quad (8)$$

## V. The Piezoelectric Polarization Modulus Set

$$[T] = [cP] [S] - [a]' [P] \quad (9)$$

$$[E] = -[a] [S] + [(\zeta)S] [P] \quad (10)$$

## VI. The Piezoelectric Polarization Constant Set

$$[S] = [sP] [T] + [b]' [P] \quad (11)$$

$$[E] = -[b] [T] + [(\zeta)T] [P] \quad (12)$$

The electric variables are connected by the relation

$$[D] = (\epsilon)s_o * [E] + [P] \quad (13)$$

where  $(\epsilon)s_o$  is the permittivity of free space, defined by

$$(\epsilon)s_o * (\mu)o * (c) * (c) = 1 ; \quad (14)$$

$(\mu)o$  is the permeability of free space, equal, by definition, to  $4 * \pi * 10^{(-7)}$ , and  $(c)$  is the velocity of light in vacuo and, also by definition, is equal exactly to  $2.99792458 * 10^8$  m/s.

From (13) the expressions for the remaining electric variables associated, respectively, with the six equation sets (1) to (12) may be found:

$$[P] = [e] [S] + [(\chi)S] [E] \quad (15)$$

$$[P] = [d] [T] + [(\chi)T] [E] \quad (16)$$

$$[P] = (\epsilon)s_o * [h] [S] + [I - (\epsilon)s_o * (\beta)S] [D] \quad (17)$$

$$[P] = (\epsilon)s_o * [g] [T] + [I - (\epsilon)s_o * (\beta)T] [D] \quad (18)$$

$$[D] = -(\epsilon)_o * [a] [S] + [I + (\epsilon)_o * (\zeta)_S] [P] \quad (19)$$

$$[D] = -(\epsilon)_o * [b] [T] + [I + (\epsilon)_o * (\zeta)_T] [P] \quad (20)$$

#### RELATIONS AMONG MATERIAL CONSTANTS

The material constants are interrelated by the following formulas:

$$[cX] [sX] = [(\epsilon)_Y] [(\beta)_Y] = [I] \quad (21)$$

$$[(\chi)_Y] [(\zeta)_Y] = [(Kr)_Y - (\chi)_Y] = [I] \quad (22)$$

In (21) and (22),  $X = E, D$ , or  $P$  and  $Y = T$  or  $S$ .

$$\begin{aligned} [cD] - [cE] &= [h]' [e] = [e]' [h] \\ &= [h]' [(\epsilon)_S] [h] = [e]' [(\beta)_S] [e] \\ &= [a]' [e - h * (\epsilon)_o] = [e - h * (\epsilon)_o]' [a] \end{aligned} \quad (23)$$

$$\begin{aligned} [cP] - [cD] &= [h]' [a] * (\epsilon)_o = [a]' [h] * (\epsilon)_o \\ &= [h]' [(\epsilon)_S] [(\zeta)_S] [h] * (\epsilon)_o \\ &= [a]' [(\beta)_S] [(\chi)_S] [a] * (\epsilon)_o \\ &= [a - h]' [e] = [e]' [a - h] \end{aligned} \quad (24)$$

$$\begin{aligned} [cP] - [cE] &= [a]' [e] = [e]' [a] \\ &= [a]' [(\chi)_S] [a] = [e]' [(\zeta)_S] [e] \\ &= [h]' [e + a * (\epsilon)_o] = [e + a * (\epsilon)_o]' [h] \end{aligned} \quad (25)$$

$$\begin{aligned} [sE] - [sD] &= [d]' [g] = [g]' [d] \\ &= [d]' [(\beta)_T] [d] = [g]' [(\epsilon)_T] [g] \\ &= [b]' [d - g * (\epsilon)_o] = [d - g * (\epsilon)_o]' [b] \end{aligned} \quad (26)$$

$$\begin{aligned} [sD] - [sP] &= [b]' [g] * (\epsilon)_o = [g]' [b] * (\epsilon)_o \\ &= [g]' [(\epsilon)_T] [(\zeta)_T] [g] * (\epsilon)_o \\ &= [b]' [(\beta)_T] [(\chi)_T] [b] * (\epsilon)_o \\ &= [b - g]' [d] = [d]' [b - g] \end{aligned} \quad (27)$$

$$\begin{aligned} [sE] - [sP] &= [b]' [d] = [d]' [b] \\ &= [b]' [(\chi)_T] [b] = [d]' [(\zeta)_T] [d] \\ &= [g]' [d + b * (\epsilon)_o] = [d + b * (\epsilon)_o]' [g] \end{aligned} \quad (28)$$

$$\begin{aligned}
[(\text{zet})S] - [(\text{zet})T] &= [b] [a]' = [a] [b]' \\
&= [b] [cP] [b]' = [a] [sP] [a]'
\end{aligned} \tag{29}$$

$$\begin{aligned}
[(\text{chi})T] - [(\text{chi})S] &= [(\text{eps})T] - [(\text{eps})S] \\
&= [e] [d]' = [d] [e]' \\
&= [d] [cE] [d]' = [e] [sE] [e]'
\end{aligned} \tag{30}$$

$$\begin{aligned}
[(\text{bet})S] - [(\text{bet})T] &= [h] [g]' = [g] [h]' \\
&= [g] [cD] [g]' = [h] [sD] [h]'
\end{aligned} \tag{31}$$

$$[e] = [d] [cE] = [(\text{eps})S] [h] = [(\text{chi})S] [a] \tag{32}$$

$$[d] = [e] [sE] = [(\text{eps})T] [g] = [(\text{chi})T] [b] \tag{33}$$

$$\begin{aligned}
[h] = [g] [cD] &= [(\text{bet})S] [e] = [(\text{chi})S] [(\text{bet})S] [a] \\
&= [I - (\text{bet})S * (\text{eps})o] [a]
\end{aligned} \tag{34}$$

$$\begin{aligned}
[g] = [h] [sD] &= [(\text{bet})T] [d] = [(\text{chi})T] [(\text{bet})T] [b] \\
&= [I - (\text{bet})T * (\text{eps})o] [b]
\end{aligned} \tag{35}$$

$$\begin{aligned}
[a] = [b] [cP] &= [(\text{zet})S] [e] = [(\text{eps})S] [(\text{zet})S] [h] \\
&= [I + (\text{zet})S * (\text{eps})o] [h]
\end{aligned} \tag{36}$$

$$\begin{aligned}
[b] = [a] [sP] &= [(\text{zet})T] [d] = [(\text{eps})T] [(\text{zet})T] [g] \\
&= [I + (\text{zet})T * (\text{eps})o] [g]
\end{aligned} \tag{37}$$

Some alternative relations are the following:

$$\begin{aligned}
[a - h] &= [(\text{zet})S] [h] * (\text{eps})o \\
&= [(\text{bet})S] [a] * (\text{eps})o
\end{aligned} \tag{38}$$

$$\begin{aligned}
[b - g] &= [(\text{zet})T] [g] * (\text{eps})o \\
&= [(\text{bet})T] [b] * (\text{eps})o
\end{aligned} \tag{39}$$

$$[e + a * (\text{eps})o] = [(\text{eps})S] [a] \tag{40}$$

$$[d + b * (\text{eps})o] = [(\text{eps})T] [b] \tag{41}$$

$$[e - h * (\text{eps})o] = [(\text{chi})S] [h] \tag{42}$$

$$[d - g * (\text{eps})o] = [(\text{chi})T] [g] \tag{43}$$

Equations (21) to (43) result from equating like dependent variables pairs selected from equations ( 1) to (12) and (15) to (20).

Each pair yields one equation in three variables, one mechanical and two electrical, or vice versa. Two other equations exist, again from (1) to (12) and (15) to (20), that contain the same three variables found in each paired equation. One of these auxiliary equations is used to eliminate one of the two variables of the same kind; the result is one equation in two variables, one electrical and one mechanical. These are now independent variables, so the coefficients must vanish; two relations between the material coefficients result. As an example, (3) and (7) both have [S] as dependent variable. Equating them produces one relation in [T], [E], and [D]; one of the electrical variables must be eliminated. This is done by using either (4) or (8); each contains the same three variables. If (8) is used to eliminate [E], one obtains  $[sE - d'g - sD][T] = [d'(\text{bet})T - g'] [D]$ . Therefore,  $[sE] - [sD] = [d]' [g]$  and  $[g] = [(\text{bet})T] [d]$ . Use of (4) instead of (8) leads to the equations  $[sE] - [sD] = [g]' [d]$  and  $[d] = [(\text{eps})T] [g]$ . There are 36 pairs, six each equating [S] and [T], and eight each equating [E], [D], and [P]. The 72 relations contain many redundancies. Relations between the elastic, piezoelectric, and dielectric constants are shown schematically in Tables 2 and 3.

#### CALCULATION SEQUENCE

Using as input [sE], [d], and [(\text{eps})T], one may compute the remaining quantities in a variety of ways. The following sequence is typical:

$$[cE] = [sE] \quad (-1) \quad (44)$$

$$[(\text{bet})T] = [(\text{eps})T] \quad (-1) \quad (45)$$

$$[e] = [d] [cE] \quad (46)$$

$$[(\text{eps})T] - [(\text{eps})S] = [e] [d]' \quad (47)$$

$$[(\text{eps})S] = [(\text{eps})T] - [e] [d]' \quad (48)$$

$$[(\text{bet})S] = [(\text{eps})S] \quad (-1) \quad (49)$$

$$[h] = [(\text{bet})S] [e] \quad (50)$$

$$[cD] - [cE] = [e]' [h] \quad (51)$$

$$[cD] = [cE] + [e]' [h] \quad (52)$$

$$[g] = [(\text{bet})T] [d] \quad (53)$$

$$[sE] - [sD] = [d]' [g] \quad (54)$$

$$[sD] = [sE] - [d]' [g] \quad (55)$$

$$[(\text{betr})S] = [(\text{bet})S] * (\text{eps})_0 \quad (56)$$

$$[(\text{zetr})S] = [(\text{betr})S] \{I - (\text{betr})S\} \quad (-1) \quad (57)$$

TABLE 2. RELATIONS AMONG MATERIAL CONSTANTS.

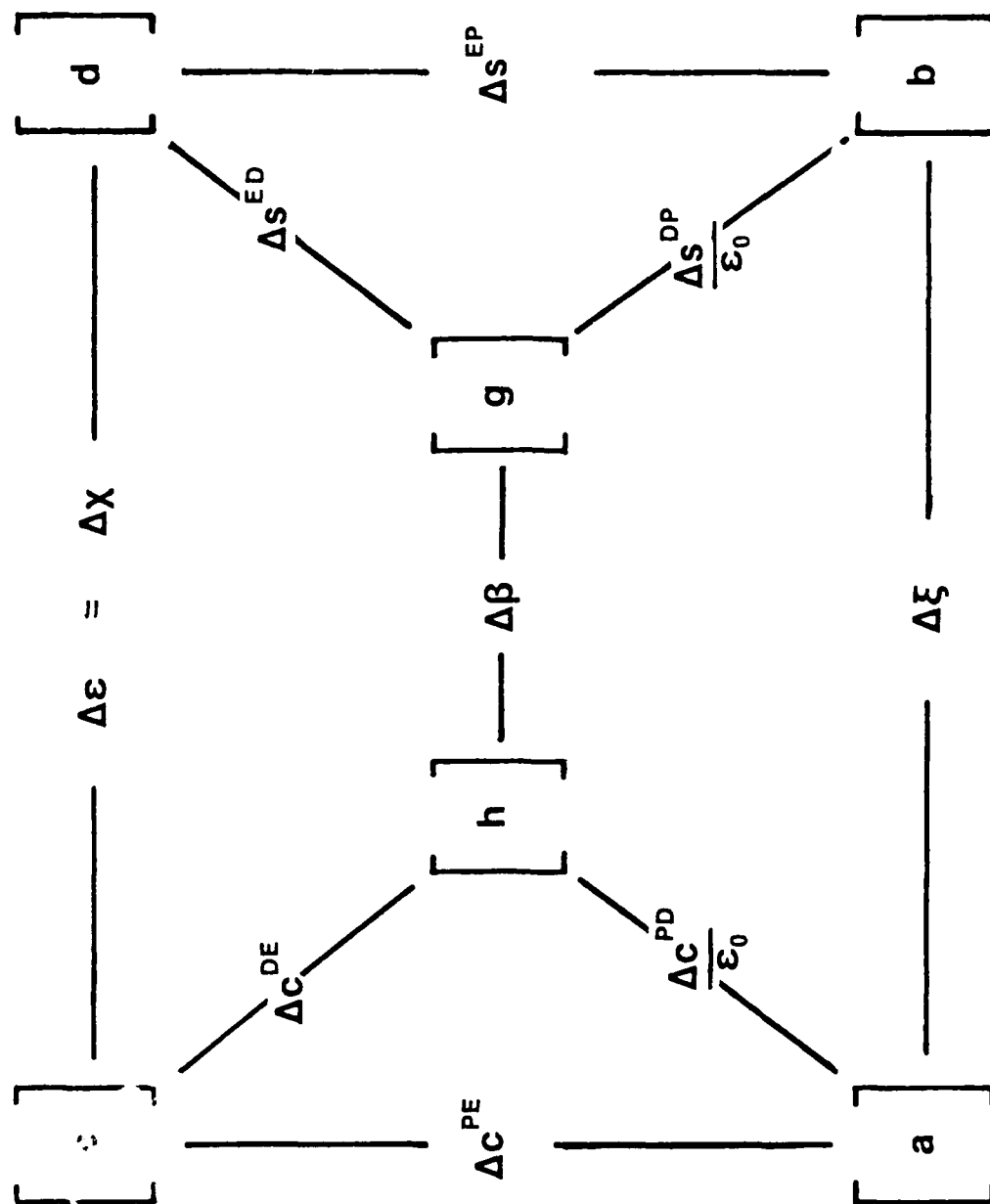
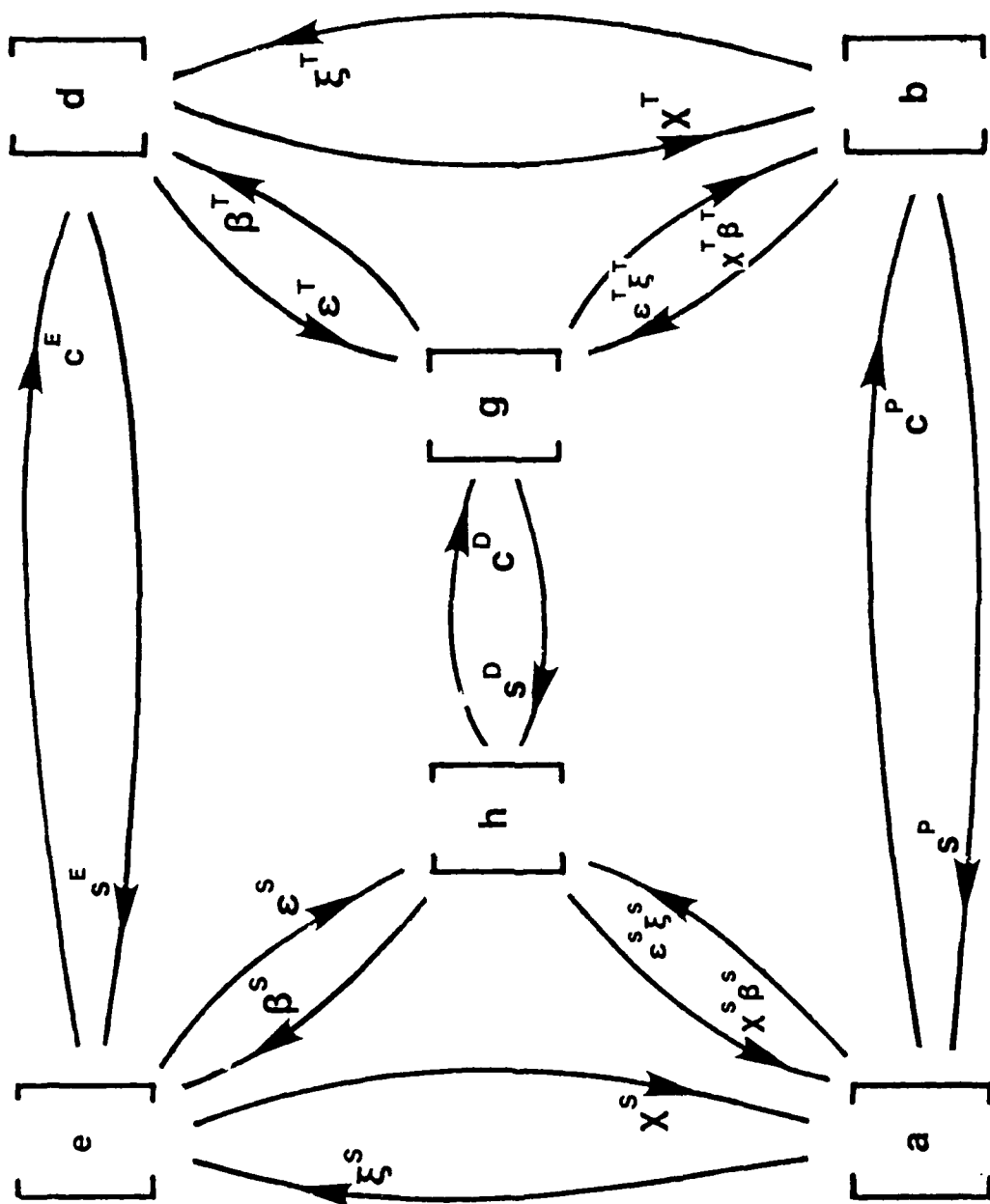


TABLE 3. FURTHER RELATIONS AMONG MATERIAL CONSTANTS.



$$[(\text{zet})S] = [(\text{zetr})] / (\text{eps})_0 \quad (58)$$

$$[(\text{betr})T] = [(\text{bet})T] * (\text{eps})_0 \quad (59)$$

$$[(\text{zetr})T] = [(\text{betr})T] [I - (\text{betr})T]^{-1} \quad (60)$$

$$[(\text{zet})T] = [(\text{zetr})T] / (\text{eps})_0 \quad (61)$$

$$[(\text{chi})S] = [(\text{zet})S]^{-1} \quad (62)$$

$$[(\text{chi})T] = [(\text{zet})T]^{-1} \quad (63)$$

$$[a] = [(\text{zet})S] [e] \quad (64)$$

$$[b] = [(\text{zet})T] [d] \quad (65)$$

$$[cP] - [cE] = [e]' [a] \quad (66)$$

$$[cP] = [cE] + [e]' [a] \quad (67)$$

$$[cP] - [cD] = [a]' [h] * (\text{eps})_0 \quad (68)$$

$$[sE] - [sP] = [d]' [b] \quad (69)$$

$$[sP] = [sE] - [d]' [b] \quad (70)$$

$$[sD] - [sP] = [g]' [b] * (\text{eps})_0 \quad (71)$$

$$[(\text{bet})S] - [(\text{bet})T] = [h] [g]' \quad (72)$$

$$[(\text{chi})T] - [(\text{chi})S] = [(\text{eps})T] - [(\text{eps})S] \quad (73)$$

$$[(\text{zet})S] - [(\text{zet})T] = [a] [b]' \quad (74)$$

A number of these relations are used as checks. For example,  $[(\text{bet})S]$  and  $[(\text{bet})T]$  are known from (45) and (49), but the difference is recomputed in (72).

#### EXPLICIT FORMULAS FOR POINT GROUP 4mm

##### Elastic:

The 6x6 elastic constant portion of Table 4 partitions into 4x4 and 2x2 submatrices. The 4x4 elastic stiffness and compliance submatrices are interrelated by formulas (75) to (93). The elastopiezodielectric matrix for class 4mm is found in Cady (Ref. 2). Other references to lithium tetraborate are given in Refs. 3 to 26.

$$A = s_{33} * (s_{11} + s_{12}) - 2 * s_{13} * s_{13} \quad (75)$$

$$B = (s_{11} - s_{12}) \quad (76)$$

$$c_{11} = +(s_{11} * s_{33} - s_{13} * s_{13}) / (A * B) \quad (77)$$

TABLE 4. ELASTOPIEZODIELECTRIC MATRICES FOR POINT GROUP 4mm.

11	12	13	00	00	00	]	00	00	31	
						]				cE ] e'
12	11	13	00	00	00	]	00	00	31	-----]-----
						]				e ](eps)S
13	13	33	00	00	00	]	00	00	33	
						]				
00	00	00	44	00	00	]	00	15	00	
						]				cD ] h'
00	00	00	00	44	00	]	15	00	00	-----]-----
						]				h ](bet)S
00	00	00	00	00	66	]	00	00	00	
-----]-----										
00	00	00	00	15	00	]	11	00	00	
						]				
00	00	00	15	00	00	]	00	11	00	cP ] a'
						]				-----]-----
31	31	33	00	00	00	]	00	00	33	a ](zet)S

Matrix entries show only subscripts.

$$c_{12} = -(s_{12} * s_{33} - s_{13} * s_{13}) / (A * B) \quad (78)$$

$$c_{13} = - s_{13} / A \quad (79)$$

$$c_{33} = (s_{11} + s_{12}) / A \quad (80)$$

$$c_{44} = 1 / s_{44} \quad (81)$$

$$c_{66} = (c_{11} - c_{12}) / 2 = s_{44} / (2 * B) \quad (82)$$

$$K = c_{33} * (c_{11} + c_{12}) - 2 * c_{13} * c_{13} \quad (83)$$

$$L = (c_{11} - c_{12}) \quad (84)$$

$$s_{11} = +(c_{11} * c_{33} - c_{13} * c_{13}) / (K * L) \quad (85)$$

$$s_{12} = -(c_{12} * c_{33} - c_{13} * c_{13}) / (K * L) \quad (86)$$

$$s_{13} = - c_{13} / K \quad (87)$$

$$s_{33} = (c_{11} + c_{12}) / K \quad (88)$$

$$s_{44} = 1 / c_{44} \quad (89)$$

$$s_{66} = 1 / c_{66} \quad (90)$$

$$\det (3 \times 3) [s] = A * B \quad (91)$$

$$\det (3 \times 3) [c] = K * L \quad (92)$$

$$A * K = B * L = A * B * K * L = 1 \quad (93)$$

Formulas (75) to (93) hold for each set of constant electrical conditions: either E, D, or P constant.

$$[CD] - [CE] = [\text{del } cDE] = [e]' [h] = [h]' [e] \quad (23)$$

$$\text{del } cDE_{11} = + e_{31} h_{31} \quad (94)$$

$$\text{del } cDE_{12} = + e_{31} h_{31} \quad (95)$$

$$\text{del } cDE_{13} = + e_{31} h_{33} = + h_{31} e_{33} \quad (96)$$

$$\text{del } cDE_{33} = + e_{33} h_{33} \quad (97)$$

$$\text{del } cDE_{44} = + e_{15} h_{15} \quad (98)$$

$$\text{del } cDE_{66} = 0 \quad (99)$$

$$\begin{aligned} [CP] - [CD] &= [\text{del } cPD] = [a]' [h] * (\epsilon)_o \\ &= [h]' [a] * (\epsilon)_o \end{aligned} \quad (24)$$

$$\text{del cPD11} = ( + a_{31} h_{31} ) * (\text{eps})_o \quad (100)$$

$$\text{del cPD12} = ( + a_{31} h_{31} ) * (\text{eps})_o \quad (101)$$

$$\begin{aligned} \text{del cPD13} &= ( + a_{31} h_{33} ) * (\text{eps})_o \\ &= ( + h_{31} a_{33} ) * (\text{eps})_o \end{aligned} \quad (102)$$

$$\text{del cPD33} = ( + a_{33} h_{33} ) * (\text{eps})_o \quad (103)$$

$$\text{del cPD44} = ( + a_{15} h_{15} ) * (\text{eps})_o \quad (104)$$

$$\text{del cPD66} = 0 \quad (105)$$

$$[cP] - [cE] = [\text{del cPE}] = [e]' [a] = [a]' [e] \quad (25)$$

$$\text{del cPE11} = + e_{31} a_{31} \quad (106)$$

$$\text{del cPE12} = + e_{31} a_{31} \quad (107)$$

$$\text{del cPE13} = + e_{31} a_{33} = + a_{31} e_{33} \quad (108)$$

$$\text{del cPE33} = + e_{33} a_{33} \quad (109)$$

$$\text{del cPE44} = + e_{15} a_{15} \quad (110)$$

$$\text{del cPE66} = 0 \quad (111)$$

From the del c13 entries we have the ratios

$$e_{31} / e_{33} = h_{31} / h_{33} = a_{31} / a_{33}. \quad (112)$$

$$[sE] - [sD] = [\text{del sED}] = [d]' [g] = [g]' [d] \quad (26)$$

$$\text{del sED11} = + d_{31} g_{31} \quad (113)$$

$$\text{del sED12} = + d_{31} g_{31} \quad (114)$$

$$\text{del sED13} = + d_{31} g_{33} = + g_{31} d_{33} \quad (115)$$

$$\text{del sED33} = + d_{33} g_{33} \quad (116)$$

$$\text{del sED44} = + d_{15} g_{15} \quad (117)$$

$$\text{del sED66} = 0 \quad (118)$$

$$\begin{aligned} [sD] - [sP] &= [g]' [b] * (\text{eps})_o \\ &= [b]' [g] * (\text{eps})_o \end{aligned} \quad (27)$$

$$\text{del sDP11} = ( + g_{31} b_{31} ) * (\text{eps})_o \quad (119)$$

$$\text{del sDP12} = ( + g_{31} b_{31} ) * (\text{eps})_o \quad (120)$$

$$\begin{aligned} \text{del sDP13} &= ( + g_{31} b_{33} ) * (\text{eps})_o \\ &= ( + b_{31} g_{33} ) * (\text{eps})_o \end{aligned} \quad (121)$$

$$\text{del sDP33} = ( + g_{33} b_{33} ) * (\text{eps})_o \quad (122)$$

$$\text{del sDP44} = ( + g_{15} b_{15} ) * (\text{eps})_o \quad (123)$$

$$\text{del sDP66} = 0 \quad (124)$$

$$[sE] - [sP] = [\text{del sEP}] = [b]' [d] = [d]' [b] \quad (28)$$

$$\text{del sEP11} = + d_{31} b_{31} \quad (125)$$

$$\text{del sEP12} = + d_{31} b_{31} \quad (126)$$

$$\text{del sEP13} = + d_{31} b_{33} = + b_{31} d_{33} \quad (127)$$

$$\text{del sEP33} = + d_{33} b_{33} \quad (128)$$

$$\text{del sEP44} = + d_{15} b_{15} \quad (129)$$

$$\text{del sEP66} = 0 \quad (130)$$

From the del s13 entries we have the ratios

$$d_{31} / d_{33} = g_{31} / g_{33} = b_{31} / b_{33}. \quad (131)$$

Piezoelectric:

$$[e] = [d] [cE] \quad (33)$$

$$e_{15} = + d_{15} cE_{44} \quad (132)$$

$$e_{31} = + d_{31} (cE_{11} + cE_{12}) + d_{33} cE_{13} \quad (133)$$

$$e_{33} = + d_{33} cE_{33} + d_{13} cE_{13} * 2 \quad (134)$$

$$[h] = [(bet)S] [e] \quad (34)$$

$$h_{15} = (bet)S_{11} e_{15} \quad (135)$$

$$h_{31} = (bet)S_{33} e_{31} \quad (136)$$

$$h_{33} = (bet)S_{33} e_{33} \quad (137)$$

$$[g] = [(bet)T] [d] \quad (35)$$

$$g_{15} = (bet)T_{11} d_{15} \quad (138)$$

$$g_{31} = (bet)T_{33} d_{31} \quad (139)$$

$$g_{33} = (bet)T_{33} d_{33} \quad (140)$$

$$[a] = [(zet)S] [e] \quad (36)$$

$$a_{15} = (zet)S_{11} e_{15} \quad (141)$$

$$a_{31} = (zet)S_{33} e_{31} \quad (142)$$

$$a_{33} = (zet)S_{33} e_{31} \quad (143)$$

$$[b] = [(zet)T] [d] \quad (37)$$

$$b_{15} = (zet)T_{11} d_{15} \quad (144)$$

$$b_{31} = (zet)T_{33} d_{31} \quad (145)$$

$$b_{33} = (zet)T_{33} d_{33} \quad (146)$$

Dielectric:

$$[(bet)Y] = [(eps)Y]^{(-1)} \quad (21)$$

$$(bet)Y_{11} = 1 / (eps)Y_{11} \quad (147)$$

$$(bet)Y_{33} = 1 / (eps)Y_{33} \quad (148)$$

$$[(zetr)Y] = [(betr)Y] [I - (betr)Y]^{(-1)} \quad (149)$$

$$(zet)Y_{11} = 1 / ((eps)Y_{11} - (eps)o) \quad (150)$$

$$(zet)Y_{33} = 1 / ((eps)Y_{33} - (eps)o) \quad (151)$$

$$[(eps)T - (eps)S] = [del (eps)] = [e] [d]' =$$

$$[(chi)T - (chi)S] = [del (chi)] = [d] [e]' \quad (30)$$

$$del (eps)_{11} = del (chi)_{11} = + e_{15} d_{15} \quad (152)$$

$$del (eps)_{33} = del (chi)_{33} = + e_{33} d_{33} + e_{31} d_{31} * 2 \quad (153)$$

$$[(bet)S - (bet)T] = [h] [g]' = [g] [h]' \quad (31)$$

$$del (bet)_{11} = + h_{15} g_{15} \quad (154)$$

$$del (bet)_{33} = + h_{33} g_{33} + h_{31} g_{31} * 2 \quad (155)$$

$$[(zet)S - (zet)T] = [del (zet)] = [a] [b]' = [b] [a]' \quad (156)$$

$$del (zet)_{11} = + a_{15} b_{15} \quad (157)$$

$$del (zet)_{33} = + a_{33} b_{33} + a_{31} b_{31} * 2 \quad (158)$$

# INPUT VALUES FOR LI2 B4 07

The values measured by Shiosaki, et al.(Ref. 1) are as follows:

TABLE 5. ISAGRIC ELASTIC COMPLIANCES.

sE11	sE12	sE13	cE33	sE44	sE66
8.81	1.23	-5.92	24.6	17.1	21.4

Units:  $10^{(-12)}$  m/N.

TABLE 6. PIEZOELECTRIC STRAIN COEFFICIENTS.

d15	d31	d33
8.07	-2.58	19.4

Units:  $10^{(-12)}$  m/v.

TABLE 7. DIELECTRIC PERMITTIVITIES AT CONSTANT STRESS.

(eps)T11	(eps)T33	(eps)T11/(eps)o	(eps)T33/(eps)o
82.61	87.92	9.33	9.93

Units:  $10^{(-12)}$  F/m.

## OUTPUT VALUES FOR LI2 B4 07

The input values from Tables 5, 6, and 7 were used to compute the remaining elastic, piezoelectric, and dielectric quantities for lithium tetraborate in the manner discussed in prior sections of this report. The results are given in Tables 8 to 15.

TABLE 8. ELASTIC STIFFNESSES.

	cE	cD	cP	del cDE	del cPE	del cPD
11	135.5	136.7	136.8	1.18	1.35	0.167
12	3.57	4.75	4.92	1.18	1.35	0.167
13	33.47	37.24	37.78	3.78	4.31	0.535
33	56.76	68.83	70.54	12.07	13.78	1.71
44	58.48	61.31	61.66	2.83	3.18	0.358
66	46.73	46.73	46.73	0.0	0.0	0.0

Units:  $10^9$  N/m<sup>2</sup>.

TABLE 9. ELASTIC COMPLIANCES.

	sE	sD	sP	del sED	del sEP	del sDP
11	8.81	8.73	8.73	0.0757	0.0842	0.00848
12	1.23	1.15	1.15	0.0757	0.0842	0.00848
13	-5.92	-5.30	-5.29	-0.569	-0.633	-0.0637
33	24.6	20.3	19.8	4.28	4.76	0.479
44	17.1	16.3	16.2	0.788	0.883	0.0946
66	24.4	21.4	21.4	0.0	0.0	0.0

Units:  $10^{-12}$  m<sup>2</sup>/N.

TABLE 10. PIEZOELECTRIC [e], [h], AND [a] VALUES.

	e	h	a
15	0.472	5.99	6.75
31	0.290	4.07	4.64
33	0.928	13.00	14.84

Units: e: C/m<sup>2</sup>; h and a:  $10^9$  V/m.

TABLE 11. PIEZOELECTRIC [d], [g], AND [b] VALUES.

	d	g	b
15	8.07	97.7	109.4
31	-2.58	-29.3	-32.6
33	19.4	220.6	245.4

Units: d:  $10^{-12}$  m/V; g and b:  $10^{-3}$  m<sup>2</sup>/C.

TABLE 12. DIELECTRIC (eps) VALUES.

	(eps)S	(eps)T	del (eps)TS
11	78.80	82.61	3.81
33	71.41	87.92	16.51

Units:  $10^{-12}$  F/m.

del (eps)TS = del (chi)TS

TABLE 13. DIELECTRIC (chi) VALUES.

	(chi)S	(chi)T	del (chi)TS
11	69.95	73.76	3.81
33	62.56	79.07	16.51

Units:  $10^{-12}$  F/m.

del (chi)TS = del (eps)TS

TABLE 14. DIELECTRIC (bet) VALUES.

	(bet)S	(bet)T	del (bet)TS
11	12.69	12.11	-0.585
33	14.00	11.37	-2.63

Units:  $10^9$  m/F.

TABLE 15. DIELECTRIC (zet) VALUES.

	(zet)S	(zet)T	del (zet)TS
11	14.30	13.56	-0.738
33	15.99	12.65	-3.34

Units:  $10(9)$  m/F.

## CONCLUSIONS

This report provides formulas interrelating the coefficients that appear in the several alternative sets of constitutive equations involving the elastic, piezoelectric, and dielectric properties of crystals. These are then specialized for crystals of class 4mm; using measured values reported for lithium tetraborate, numerical values of the elements of the polarization matrices are calculated.

## REFERENCES

1. T. Shiosaki, M. Adachi, and A. Kawabata, "Growth and Properties of Piezoelectric Lithium Tetraborate Crystal for BAW and SAW Devices," IEEE Intl. Symp. Applications Ferroelectrics, Lehigh University, June 1986, pp. 455-464.
2. W. G. Cady, Piezoelectricity, McGraw-Hill, New York, 1946; Dover, New York, 1964.
3. G. S. Smith and G. E. Rindone, "High-Temperature Energy Relations in the Alkali Borates: Binary Alkali Borate Compounds and Their Glasses," J. Am. Ceram. Soc., Vol. 44, No. 2, 1961, pp. 72-78.
4. J. Krogh-Moe, "The Crystal Structure of Lithium Diborate,  $\text{Li}_2\text{O} \cdot 2\text{B}_2\text{O}_3$ ," Acta Cryst., Vol. 15, 1962, pp. 190-193.
5. J. Krogh-Moe, Acta Cryst., Vol. B24, 1968, pp. 179ff.
6. S. R. Nagel, L. W. Herron, and C. G. Bergeron, "Crystal Growth of  $\text{Li}_2\text{B}_4\text{O}_7$ ," J. Am. Ceram. Soc., Vol. 60, No. 3-4, March-April 1977, pp. 172-173.
7. J. D. Garrett, M. N. Iyer, J. E. Greedan, "The Czochralski Growth of  $\text{LiBO}_2$  and  $\text{Li}_2\text{B}_4\text{O}_7$ ," J. Cryst. Growth, Vol. 41, 1977, pp. 225-227.
8. R. W. Whatmore, N. M. Shorrocks, C. O'Hara, F. W. Ainger, and I. M. Young, "Lithium Tetraborate: A New Temperature-Compensated SAW Substrate Material," Elx. Lett., Vol. 17, No. 1, 8th January 1981, pp. 11-12.
9. N. M. Shorrocks, R. W. Whatmore, F. W. Ainger, and I. M. Young, "Lithium Tetraborate: A New Temperature Compensated Piezoelectric Substrate Material for Surface Acoustic Wave Devices," IEEE Ultrasonics Symp. Proc., 1981, pp. 337-340.
10. B. Lewis, N. M. Shorrocks, and R. W. Whatmore, "An Assessment of Lithium Tetraborate for SAW Applications," IEEE Ultrasonics Symp. Proc., 1982, pp. 389-393.
11. D. S. Robertson and I. M. Young, "The Growth and Growth Mechanism of Lithium Tetraborate," J. Mater. Sci., Vol. 17, 1982, pp. 1729-1738.
12. Y. Ebata, H. Suzuki, S. Matsumura, and K. Fukuta, "SAW Propagation Characteristics on  $\text{Li}_2\text{B}_4\text{O}_7$ ," Proc. 3rd Symp. Ultrasonic Elx., Tokyo, 1982; Jpn. J. Appl. Phys., Vol. 22, 1983, Suppl. 22-3, pp. 160-162.
13. C. D. J. Emin and J. F. Werner, "The Bulk Acoustic Wave Properties of Lithium Tetraborate," Proc. 37th Annual Frequency Control Symp., June 1983, pp. 136-143.

14. R. C. Peach, C. D. J. Emin, J. F. Werner, and S. P. Doherty, "High Coupling Piezoelectric Resonators Using Lithium Tetraborate," IEEE Ultrasonics Symp. Proc., 1983, pp. 521-526.
15. K. Fukuta, J. Ushizawa, H. Suzuki, Y. Ebata, and S. Matsumura, "Growth and Properties of  $\text{Li}_2\text{B}_4\text{O}_7$  Single Crystal for SAW Device Applications," Proc. 4th Meeting on Ferroelectric Materials and Their Applications, Kyoto, 1983; Jpn. J. Appl. Phys., Vol. 22, 1983, Suppl. 22-2, pp. 140-142.
16. T. Shiosaki, M. Adachi, H. Kobayashi, K. Araki, and A. Kawabata, "Elastic, Piezoelectric, Acousto-Optic and Electro-Optic Properties of  $\text{Li}_2\text{B}_4\text{O}_7$ ," Proc. 5th Symp. Ultrasonic Elx., Tokyo, 1984; Jpn. J. Appl. Phys., Vol. 24, 1985, Suppl. 24-1, pp. 25-27.
17. H. Suzuki, Y. Ebata, S. Matsumura, and J. Ushizawa, "Surface Acoustic Wave Device," U.S. Patent 4,672,255, issued June 9, 1987.
18. M. Adachi, T. Shiosaki, and A. Kawabata, "Crystal Growth of Lithium Tetraborate ( $\text{Li}_2\text{B}_4\text{O}_7$ )," Proc. 5th Meeting on Ferroelectric Materials and Their Applications, Kyoto, 1985; Jpn. J. Appl. Phys., Vol. 24, 1985, Suppl. 24-3, pp. 72-75.
19. Y. Fujiwara, M. Ono, M. Sakai, and N. Wakatsuki, "Strip Type Resonator of Lithium Tetraborate," Proc. 39th Annual Frequency Control Symp., May 1985, pp. 351-355.
20. M. Adachi, T. Shiosaki, H. Kobayashi, O. Ohnishi, and A. Kawabata, "Temperature Compensated Piezoelectric Lithium Tetraborate Crystal for High Frequency Surface Acoustic Wave and Bulk Wave Device Applications," IEEE Ultrasonics Symp. Proc., 1985, pp. 228-232.
21. A. S. Bhalla, L. E. Cross, and R. W. Whatmore, "Pyroelectric and Piezoelectric Properties of Lithium Tetraborate Single Crystal," Jpn. J. Appl. Phys., Vol. 24, 1985, Suppl. 24-2, pp. 727-729.
22. A. Ballato, E. R. Hatch, T. Lukaszek, and M. Mizan, "Lateral-Field Coupling of Rotated BAW Plates with 3m, 4mm, &  $\bar{4}3\text{m}$  Symmetries," IEEE Ultrasonics Symp. Proc., 1986, pp. 339-342.
23. Y. Ebata and M. Koshino, "SAW Resonator and Resonator Filter on  $\text{Li}_2\text{B}_4\text{O}_7$  Substrate," Jpn. J. Appl. Phys., Vol. 26, 1987, Suppl. 26-1, pp. 123-125.
24. H. Abe, H. Saitou, M. Ohmura, T. Yamada, and K. Miwa, "Lithium Tetraborate ( $\text{Li}_2\text{B}_4\text{O}_7$ ) SAW Resonators," IEEE Ultrasonics Symp. Proc. 1987, pp. 91-94.

26. S. Matsumura, T. Omi, N. Yamaji, and Y. Ebata, "A  $45^\circ$  X Cut  $\text{Li}_2\text{B}_4\text{O}_7$  Single Crystal Substrate for SAW Resonators," IEEE Ultrasonics Symp. Proc., 1987, pp. 247-250.

ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY  
MANDATORY DISTRIBUTION LIST  
CONTRACT OR IN-HOUSE TECHNICAL REPORTS

15 Nov 88  
Page 1 of 2

101 Defense Technical Information Center\*  
ATTN: DTIC-FDAC  
Cameron Station (Bldg 5)  
Alexandria, VA 22304-6145 (\*Note: Two copies for DTIC will  
be sent from STINFO Office.)

483 Director  
US Army Material Systems Analysis Actv  
ATTN: DRXSY-MP  
001 Aberdeen Proving Ground, MD 21005

563 Commander, AMC  
ATTN: AMCDE-SC  
5001 Eisenhower Ave.  
001 Alexandria, VA 22333-0001

609 Commander, LABCOM  
ATTN: AMSLC-CG, CD, CS (In turn)  
2800 Powder Mill Road  
001 Adelphi, Md 20783-1145

612 Commander, LABCOM  
ATTN: AMSLC-CT  
2800 Powder Mill Road  
001 Adelphi, MD 20783-1145

680 Commander,  
US Army Laboratory Command  
Fort Monmouth, NJ 07703-5000  
1 - SLCET-DD  
2 - SLCET-DT (M. Howard)  
1 - SLCET-DB  
35 - Originating Office

681 Commander, CECOM  
R&D Technical Library  
Fort Monmouth, NJ 07703-5000  
1 - ASQNC-ELC-I-T (Tech Library)  
3 - ASQNC-ELC-I-T (STINFO)

705 Advisory Group on Electron Devices  
201 Varick Street, 9th Floor  
002 New York, NY 10014-4877

ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY  
SUPPLEMENTAL CONTRACT DISTRIBUTION LIST  
(ELECTIVE)

15 Nov 88  
Page 2 of 2

205	Director Naval Research Laboratory ATTN: CODE 2627 001 Washington, DC 20375-5000	603	Cdr, Atmospheric Sciences Lab LABCOM ATTN: SLCAS-SY-S 001 White Sands Missile Range, NM 88002
221	Cdr, PM JTFUSION ATTN: JTF 1500 Planning Research Drive 001 McLean, VA 22102	607	Cdr, Harry Diamond Laboratories ATTN: SLCHD-CO, TD (In turn) 2800 Powder Mill Road 001 Adelphi, MD 20783-1145
301	Rome Air Development Center ATTN: Documents Library (TILD) 001 Griffiss AFB, NY 13441		
437	Deputy for Science & Technology Office, Asst Sec Army (R&D) 001 Washington, DC 20310		
438	HQDA (DAMA-ARZ-D/Dr. F.D. Verderame) 001 Washington, DC 20310		
520	Dir, Electronic Warfare/Reconnaissance Surveillance and Target Acquisition Ctr ATTN: AMSEL-EW-D 001 Fort Monmouth, NJ 07703-5000		
523	Dir, Reconnaissance Surveillance and Target Acquisition Systems Directorate ATTN: AMSEL-EW-DR 001 Fort Monmouth, NJ 07703-5000		
524	Cdr, Marine Corps Liaison Office ATTN: AMSEL-LN-MC 001 Fort Monmouth, NJ 07703-5000		
564	Dir, US Army Signals Warfare Ctr ATTN: AMSEL-SW-OS Vint Hill Farms Station 001 Warrenton, VA 22186-5100		
602	Dir, Night Vision & Electro-Optics Ctr CECOM ATTN: AMSEL-NV-D 001 Fort Belvoir, VA 22060-5677		